

Mathematics is life!

In[\*]:= **4 \* 8**

Out[\*]=  
32

In[\*]:= **3 + 6 \* 5**

Out[\*]=  
33

In[\*]:= **274 / 3**

Out[\*]=  
 $\frac{274}{3}$

In[\*]:= **274 / 3.**

Out[\*]=  
91.3333

In[\*]:= **(3.24 \* 6.791 - 14.7) / (4.5 + 82 / 3)**

Out[\*]=  
0.229409

In[\*]:= **[(3.24 \* 6.791) - 14.7] / (4.5 + 82 / 3)**

The only grouping symbol is parenthesis!

In[\*]:= **2.6^2.99**

Out[\*]=  
17.4089

In[\*]:= **3 × 5**

Out[\*]=  
15

To indicate multiplication a blank space between 3 and 5 is needed.

In[\*]:= **Log[12.7]**

Out[\*]=  
2.5416

```
In[*]:=
```

```
E^2.5
```

```
Out[*]=
```

```
12.1825
```

```
In[*]:=
```

```
Log[E, 12.7]
```

```
Out[*]=
```

```
2.5416
```

```
In[*]:=
```

```
Log[10, 72.8]
```

```
Out[*]=
```

```
1.86213
```

```
In[*]:=
```

```
Sqrt[64.0]
```

```
Out[*]=
```

```
8.
```

```
In[*]:=
```

```
Abs[-2.5]
```

```
Out[*]=
```

```
2.5
```

```
In[*]:=
```

```
I * I
```

```
Out[*]=
```

```
-1
```

```
In[*]:=
```

```
E
```

```
Out[*]=
```

```
e
```

```
In[*]:=
```

```
N[E]
```

```
Out[*]=
```

```
2.71828
```

```
In[*]:=
```

```
Pi
```

```
Out[*]=
```

```
 $\pi$ 
```

```
In[*]:=
```

```
N[Pi]
```

```
Out[*]=
```

```
3.14159
```

```
In[ ]:=
```

```
Pi // N
```

```
Out[ ]=
```

```
3.14159
```

```
In[ ]:=
```

```
NumberForm[N[E], 10]
```

```
Out[ ]//NumberForm=
```

```
2.718281828
```

```
In[ ]:=
```

```
NumberForm[N[Pi], 16]
```

```
Out[ ]//NumberForm=
```

```
3.141592653589793
```

```
In[ ]:=
```

```
N[Pi, 17]
```

```
Out[ ]=
```

```
3.1415926535897932
```

```
In[ ]:=
```

```
N[Pi, 1000]
```

```
Out[ ]=
```

```

3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862803482
534211706798214808651328230664709384460955058223172535940812848111745028410270193852110555
964462294895493038196442881097566593344612847564823378678316527120190914564856692346034861
045432664821339360726024914127372458700660631558817488152092096282925409171536436789259036
001133053054882046652138414695194151160943305727036575959195309218611738193261179310511854
807446237996274956735188575272489122793818301194912983367336244065664308602139494639522473
719070217986094370277053921717629317675238467481846766940513200056812714526356082778577134
275778960917363717872146844090122495343014654958537105079227968925892354201995611212902196
086403441815981362977477130996051870721134999999837297804995105973173281609631859502445945
534690830264252230825334468503526193118817101000313783875288658753320838142061717766914730
359825349042875546873115956286388235378759375195778185778053217122680661300192787661119590
9216420199

```

If a palette is not open, you can get it by clicking consecutively on the buttons Palettes, and Basic Math Assistant.

```
In[ ]:=
```

```
35.2
```

```
Out[ ]=
```

```
302.713
```

```
In[ ]:=
```

```
 $\sqrt{37.4}$ 
```

```
Out[ ]=
```

```
6.11555
```

In[\*]:=  $\sqrt{3.1^{2.7}}$

Out[\*]=  
4.60615

In[\*]:=  $\frac{125.3}{72}$

Out[\*]=  
1.74028

In[\*]:=  $N\left[\sqrt{\frac{5}{9}}\right]$

Out[\*]=  
0.745356

In[\*]:=  $N[E^2]$

Out[\*]=  
7.38906

In[\*]:=  $\text{Log}[\%]$

Out[\*]=  
2.

This was ln 2.

In[\*]:=  $\text{Simplify}[3x^2 - x - 9 + x^2 + 7x + 5]$

Out[\*]=  
 $-4 + 6x + 4x^2$

In[\*]:=  $\text{Cancel}[(x^2 - 2x - 3) / (x^2 - 9)]$

Out[\*]=  
 $\frac{1+x}{3+x}$

In[\*]:=  $\text{Factor}[x^4 - 1]$

Out[\*]=  
 $(-1+x)(1+x)(1+x^2)$

In[\*]:=  $\text{Factor}[a s + b a s]$

Out[\*]=  
 $a(1+b)s$

```
In[ ]:=  
Expand[(x + 2)^3]
```

```
Out[ ]:=  
8 + 12 x + 6 x^2 + x^3
```

```
In[ ]:=  
Apart[x / ((x - 2) (x^2 + 3))]
```

```
Out[ ]:=  

$$\frac{2}{7(-2+x)} + \frac{3-2x}{7(3+x^2)}$$

```

```
In[ ]:=  
Together[x / (x + 5) - 1 / (x - 4)]
```

```
Out[ ]:=  

$$\frac{-5 - 5x + x^2}{(-4+x)(5+x)}$$

```

```
In[ ]:=  
x = 5; y = 12; z = a + b;
```

```
In[ ]:=  
x y^2
```

```
Out[ ]:=  
720
```

To indicate multiplication a blank space between **x** and **y^2** is needed. Or, one can use the multiplication sign **\***.

```
In[ ]:=  
Expand[z^x]
```

```
Out[ ]:=  
a^5 + 5 a^4 b + 10 a^3 b^2 + 10 a^2 b^3 + 5 a b^4 + b^5
```

```
In[ ]:=  
Clear[x, y, z]
```

```
In[ ]:=  
Solve[3 x - 8 == 4]
```

```
Out[ ]:=  
{ {x -> 4} }
```

```
In[ ]:=  
Solve[a x - 2 == 3 a, x]
```

```
Out[ ]:=  
{ {x ->  $\frac{2+3a}{a}$  } }
```

```
In[ ]:=  
Solve[{x - 2 y == 4, x - 1 == 5 y}, {x, y}]
```

```
Out[ ]:=  
{ {x -> 6, y -> 1} }
```

In[\*]:=

**NSolve**[ $2x - 3 == x^2 - 3x - 4$ ]

Out[\*]=

{ $x \rightarrow -0.192582$ }, { $x \rightarrow 5.19258$ }

In[\*]:=

**NSolve**[ $\{x^2 + y^2 == 16, y == x^2 - 2x + 2\}, \{x, y\}$ ]

Out[\*]=

{ $x \rightarrow 2.46607, y \rightarrow 3.14936$ }, { $x \rightarrow 1.12368 - 2.35752 i, y \rightarrow -4.54261 - 0.583168 i$ },  
{ $x \rightarrow 1.12368 + 2.35752 i, y \rightarrow -4.54261 + 0.583168 i$ }, { $x \rightarrow -0.713436, y \rightarrow 3.93586$ }

In[\*]:=

**FindRoot**[ $\text{Cos}[x] == x^3, \{x, 1\}$ ]

Out[\*]=

{ $x \rightarrow 0.865474$ }

In[\*]:=

**f**[x\_] :=  $x^2 - 5$ ; **g**[x\_] :=  $\text{Log}[x] / x$

In[\*]:=

**f**[3]

Out[\*]=

4

In[\*]:=

**g**[**f**[x]]

Out[\*]=

$$\frac{\text{Log}[-5 + x^2]}{-5 + x^2}$$

In[\*]:=

**g**[**f**[3]]

Out[\*]=

$$\frac{\text{Log}[4]}{4}$$

In[\*]:=

**N**[**g**[**f**[3]]]

Out[\*]=

0.346574

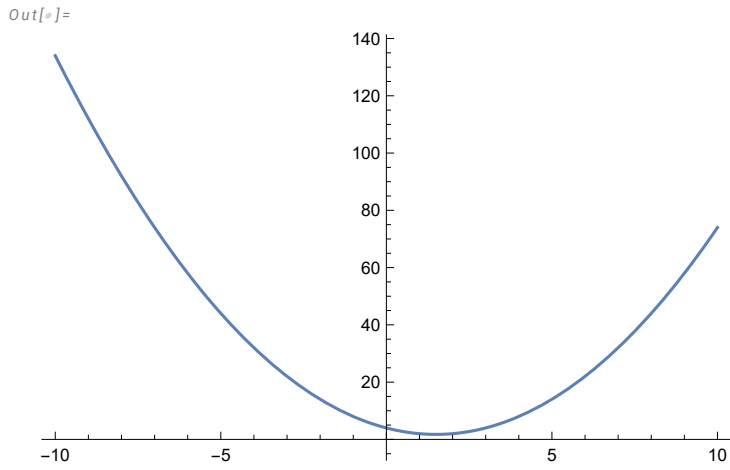
In[\*]:=

**Solve**[ $f[x] == 4$ ]

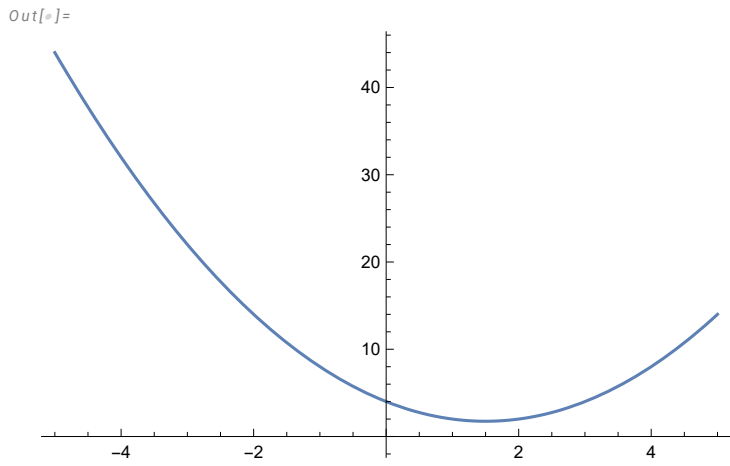
Out[\*]=

{ $x \rightarrow -3$ }, { $x \rightarrow 3$ }

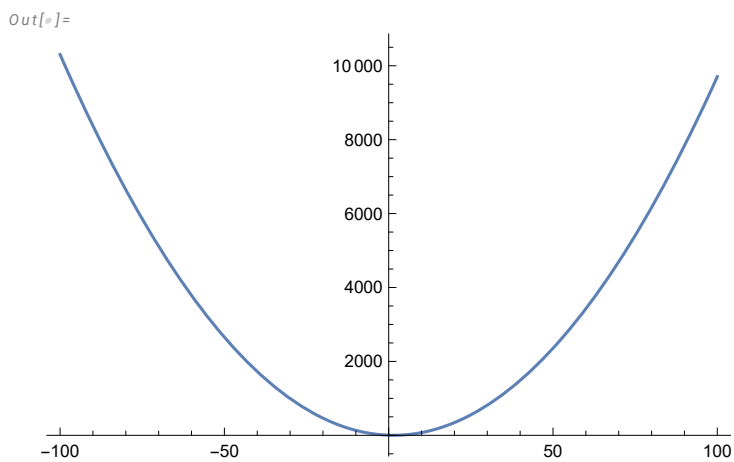
```
In[ ]:=  
Plot[x^2 - 3 x + 4, {x, -10, 10}]
```



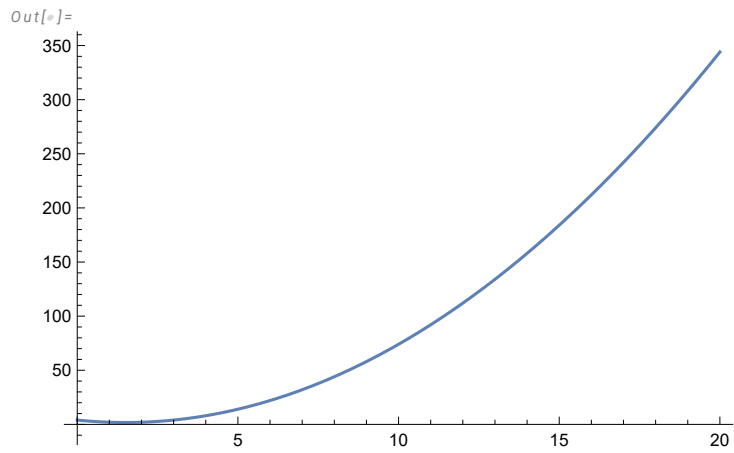
```
In[ ]:=  
Plot[x^2 - 3 x + 4, {x, -5, 5}]
```



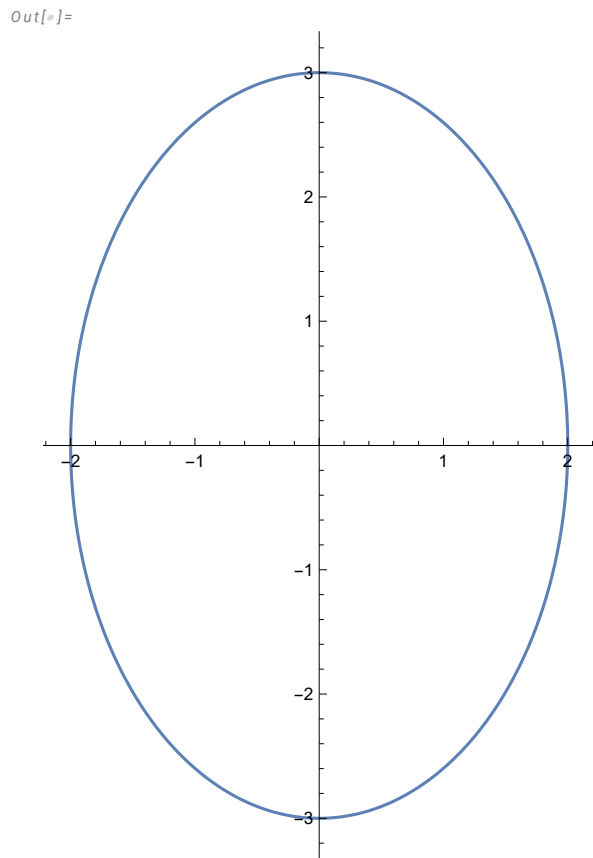
```
In[ ]:=  
Plot[x^2 - 3 x + 4, {x, -100, 100}]
```



```
In[ ]:=  
Plot[x^2 - 3 x + 4, {x, 0, 20}]
```

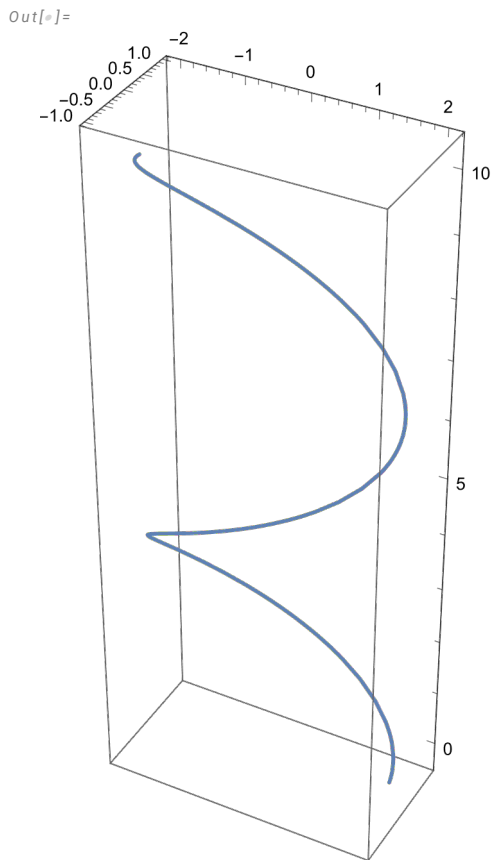


```
In[ ]:=  
ParametricPlot[{2 Cos[t], 3 Sin[t]}, {t, 0, 2 Pi}]
```

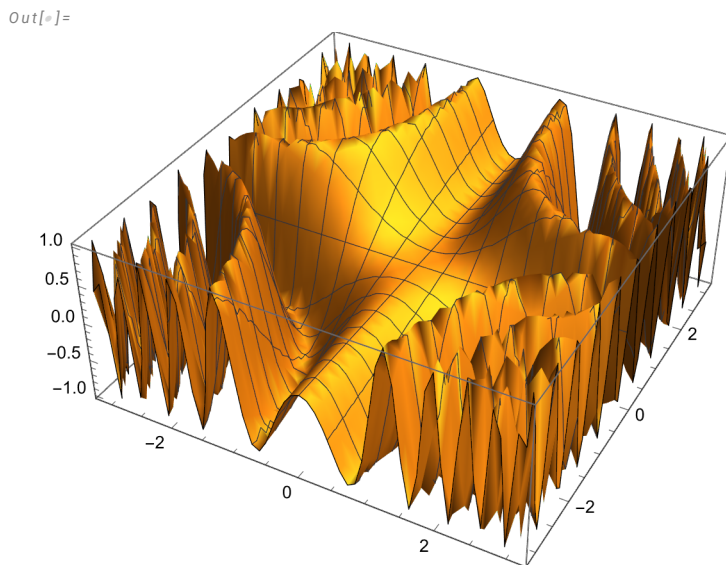




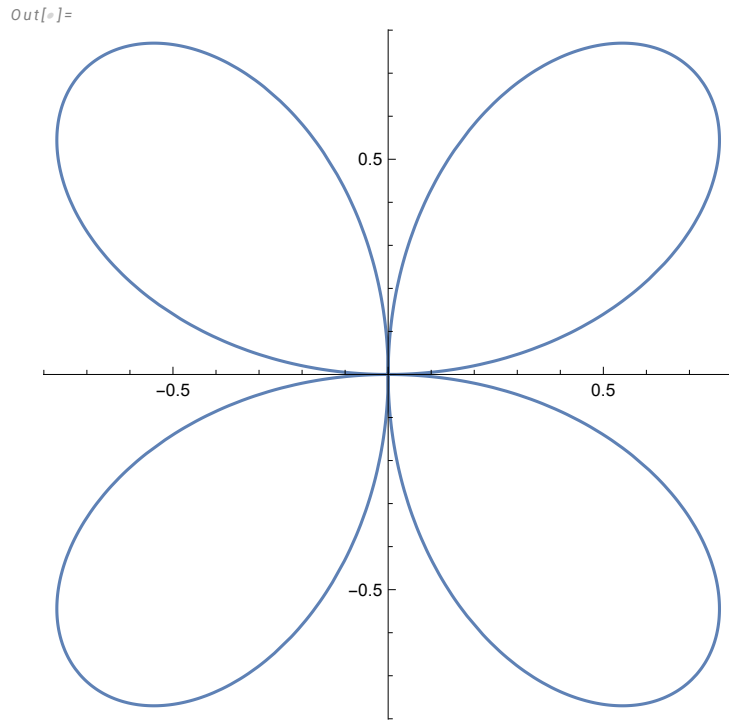
```
In[*]:= ParametricPlot3D[{2 Cos[t], Sin[t], t}, {t, 0, 10}]
```



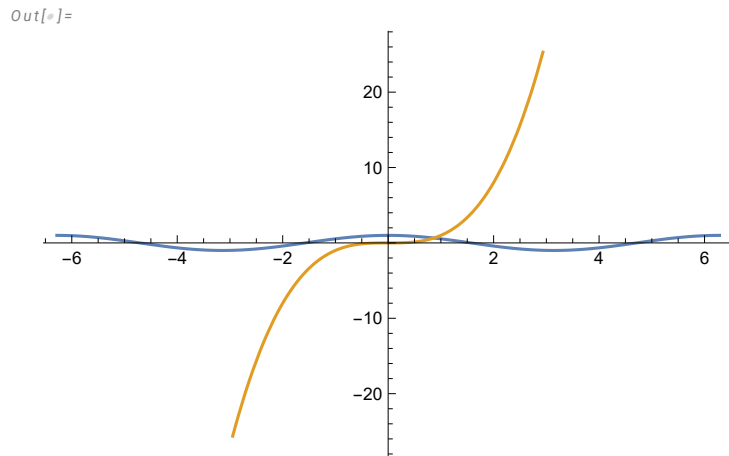
```
In[*]:= Plot3D[Sin[x^2 y], {x, -Pi, Pi}, {y, -Pi, Pi}]
```



```
In[*]:= PolarPlot[Sin[2 t], {t, 0, 2 Pi}]
```



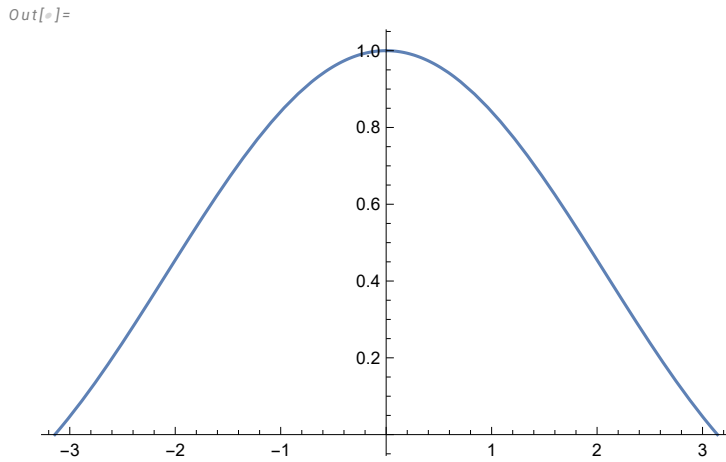
```
In[*]:= Plot[{Cos[x], x^3}, {x, -2 Pi, 2 Pi}]
```



```
In[*]:= FindRoot[Cos[x] == x^3, {x, 0.5}]
```

```
Out[*]= {x -> 0.865474}
```

```
In[*]:=
Plot[Sin[x] / x, {x, -Pi, Pi}]
```



```
In[*]:=
Limit[Sin[x] / x, x → 0, Direction → "FromBelow"]
```

Out[\*]=  
1

```
In[*]:=
Limit[Sin[x] / x, x → 0, Direction → "FromAbove"]
```

Out[\*]=  
1

```
In[*]:=
Limit[Sin[x] / x, x → 0]
```

Out[\*]=  
1

**Verify existence of limits!**

```
In[*]:=
D[x^2 Sin[x] - 3 x + 1, x]
```

Out[\*]=  
 $-3 + x^2 \cos[x] + 2 x \sin[x]$

```
In[*]:=
f[x_] := x^3 - 2 x^2 + 5; f'[x]
```

Out[\*]=  
 $-4 x + 3 x^2$

```
In[*]:=
f''[x]
```

Out[\*]=  
 $-4 + 6 x$

In[\*]:=

**D[f[x], {x, 3}]**

Out[\*]=

6

In[\*]:=

**D[x^2 y[x]^2 + x Sin[y[x]] == 1, x]**

Out[\*]=

**Sin[y[x]] + 2 x y[x]^2 + x Cos[y[x]] y'[x] + 2 x^2 y[x] y'[x] == 0**

In[\*]:=

**Solve[%, y'[x]]**

Out[\*]=

**{ {y'[x] →  $\frac{-\text{Sin}[y[x]] - 2 x y[x]^2}{x (\text{Cos}[y[x]] + 2 x y[x])}$  } }**

In[\*]:=

**f[x\_, y\_, z\_] := x^4 z^3 y + x Sin[z + y];**  
**D[f[x, y, z], x]**

Out[\*]=

**4 x^3 y z^3 + Sin[y + z]**

In[\*]:=

**D[f[x, y, z], {x, 2}, {y, 1}, {z, 1}]**

Out[\*]=

**36 x^2 z^2**

In[\*]:=

**Integrate[x^2 + 1, x]**

Out[\*]=

**x +  $\frac{x^3}{3}$**

In[\*]:=

**Integrate[x^2 + 1, {x, -1, 2}]**

Out[\*]=

6

In[\*]:=

**f[x\_, y\_] := 1 - (x^2 / 4) - (y^2 / 9);**  
**Integrate[f[x, y], {x, -2, 2}, {y, -3, 3}]**

Out[\*]=

8

In[\*]:=

**Integrate[Exp[-x^2], {x, 0, Infinity}]**

Out[\*]=

**$\frac{\sqrt{\pi}}{2}$**

```
In[*]:=
Integrate[3 x^2 (2010 - x^3)^1999, x]
```

```
Out[*]=
```

3 ( ... 1 ... )

large output    show less    show more    show all    set size limit...

You can easily find the above integral using u substitution:  $-\frac{1}{2000} (2010 - x^3)^{2000} + C.$

```
In[*]:=
Series[E^x, {x, 0, 6}]
```

```
Out[*]=
```

$$1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \frac{x^6}{720} + O[x]^7$$

```
In[*]:=
```

```
Normal[Series[Cos[x], {x, 0, 8}]]
```

```
Out[*]=
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320}$$

```
In[*]:=
```

```
Series[Log[x], {x, 1, 5}]
```

```
Out[*]=
```

$$(x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4 + \frac{1}{5}(x-1)^5 + O[x-1]^6$$

```
In[*]:=
```

```
Sum[1/n^2, {n, 1, Infinity}]
```

```
Out[*]=
```

$$\frac{\pi^2}{6}$$

```
In[*]:=
```

```
u = {1, -3}
```

```
Out[*]=
```

```
{1, -3}
```

```
In[*]:=
```

```
MatrixForm[u]
```

```
Out[*]//MatrixForm=
```

$$\begin{pmatrix} 1 \\ -3 \end{pmatrix}$$

```
In[*]:=
```

```
v = {{-2, 5, -1}}
```

```
Out[*]=
```

```
{{-2, 5, -1}}
```

In[\*]:=

**MatrixForm[v]**

Out[\*]//MatrixForm=

$(-2 \ 5 \ -1)$

In[\*]:=

**m = {{1, 2}, {3, 4}, {5, 6}}**

Out[\*]=

$\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$

In[\*]:=

**n = {{3, 2, 1}, {6, 5, 4}}**

Out[\*]=

$\{\{3, 2, 1\}, \{6, 5, 4\}\}$

In[\*]:=

**l = {{3, -1}, {-2, 6}, {-4, 5}}**

Out[\*]=

$\{\{3, -1\}, \{-2, 6\}, \{-4, 5\}\}$

In[\*]:=

**m - 2 l**

Out[\*]=

$\{\{-5, 4\}, \{7, -8\}, \{13, -4\}\}$

In[\*]:=

**n.m**

Out[\*]=

$\{\{14, 20\}, \{41, 56\}\}$

In[\*]:=

**{{14, 20}, {41, 56}}**

Out[\*]=

$\{\{14, 20\}, \{41, 56\}\}$

In[\*]:=

**MatrixForm[n.m]**

Out[\*]//MatrixForm=

$\begin{pmatrix} 14 & 20 \\ 41 & 56 \end{pmatrix}$

In[\*]:=

**Dimensions[m]**

Out[\*]=

$\{3, 2\}$

In[\*]:=

**i = IdentityMatrix[3]**

Out[\*]=

$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$

In[\*]:=

**d = DiagonalMatrix[{2, 2, -3}]**

Out[\*]=

**{{2, 0, 0}, {0, 2, 0}, {0, 0, -3}}**

In[\*]:=

**Transpose[m]**

Out[\*]=

**{{1, 3, 5}, {2, 4, 6}}**

In[\*]:=

**MatrixForm[Transpose[m]]**

Out[\*]//MatrixForm=

$$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$$

In[\*]:=

**a = {{2., 3, -5}, {-1, 4, 2}, {5, 7, 2}}**

Out[\*]=

**{{2., 3, -5}, {-1, 4, 2}, {5, 7, 2}}**

In[\*]:=

**Det[a]**

Out[\*]=

**159.**

In[\*]:=

**Inverse[a]**

Out[\*]=

**{{-0.0377358, -0.257862, 0.163522},  
{0.0754717, 0.18239, 0.00628931}, {-0.169811, 0.00628931, 0.0691824}}**

In[\*]:=

**MatrixForm[Inverse[a]]**

Out[\*]//MatrixForm=

$$\begin{pmatrix} -0.0377358 & -0.257862 & 0.163522 \\ 0.0754717 & 0.18239 & 0.00628931 \\ -0.169811 & 0.00628931 & 0.0691824 \end{pmatrix}$$

In[\*]:=

**MatrixPower[a, 3]**

Out[\*]=

**{{-101., -238., 58.}, {66., 239., 68.}, {-66., 218., 35.}}**

In[\*]:=

**Eigenvalues[a]**

Out[\*]=

**{6.52411 + 0. i, 0.737944 + 4.88125 i, 0.737944 - 4.88125 i}**

In[\*]:=

**Eigenvalues[a]**

Out[\*]=

{ $\{-0.312408 + 0. \text{i}, 0.662901 + 0. \text{i}, 0.680414 + 0. \text{i}\}$ ,  
 $\{0.746572 + 0. \text{i}, -0.12497 + 0.189974 \text{i}, 0.113461 - 0.614857 \text{i}\}$ ,  
 $\{0.746572 + 0. \text{i}, -0.12497 - 0.189974 \text{i}, 0.113461 + 0.614857 \text{i}\}$ }

In[\*]:=

**Eigensystem[a]**

Out[\*]=

{ $\{6.52411, 0.737944 + 4.88125 \text{i}, 0.737944 - 4.88125 \text{i}\}$ ,  
 $\{-0.312408 + 0. \text{i}, 0.662901 + 0. \text{i}, 0.680414 + 0. \text{i}\}$ ,  
 $\{0.746572 + 0. \text{i}, -0.12497 + 0.189974 \text{i}, 0.113461 - 0.614857 \text{i}\}$ ,  
 $\{0.746572 + 0. \text{i}, -0.12497 - 0.189974 \text{i}, 0.113461 + 0.614857 \text{i}\}$ }

In[\*]:=

**Solve**[ $\{2x + 3y - 5z == 14, -x + 4y + 2z == 0, 5x + 7y + 2z == -9\}$ ,  $\{x, y, z\}$ ]

Out[\*]=

{ $\{x \rightarrow -2, y \rightarrow 1, z \rightarrow -3\}$ }

In[\*]:=

**b = {14, 0, -9}**

Out[\*]=

{**14, 0, -9**}

In[\*]:=

**Inverse[a].b**

Out[\*]=

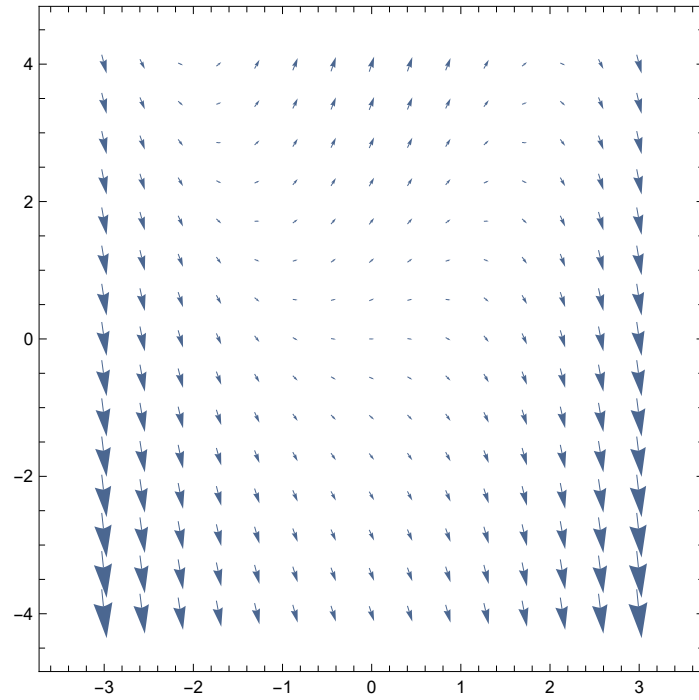
{**-2., 1., -3.**}



```
In[ ]:=
```

```
VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

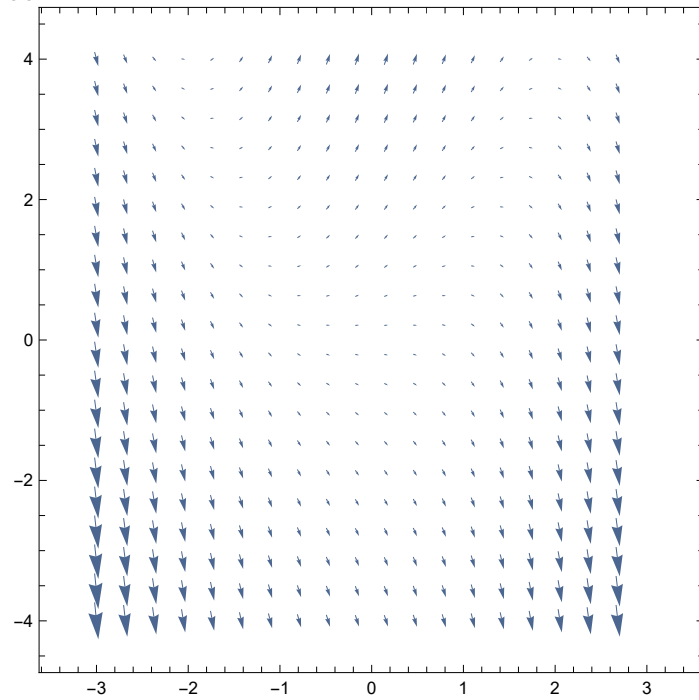
```
Out[ ]:=
```



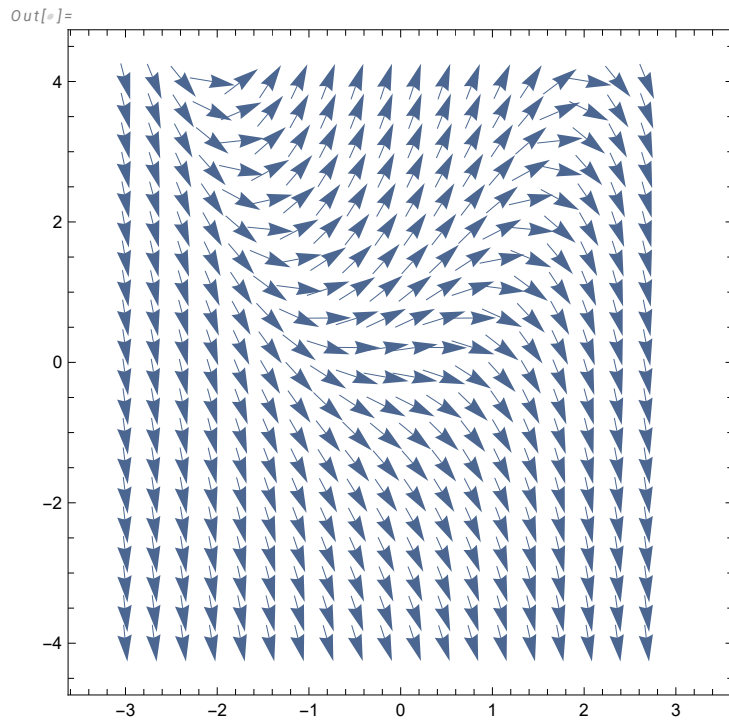
```
In[ ]:=
```

```
VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}, VectorPoints -> 20]
```

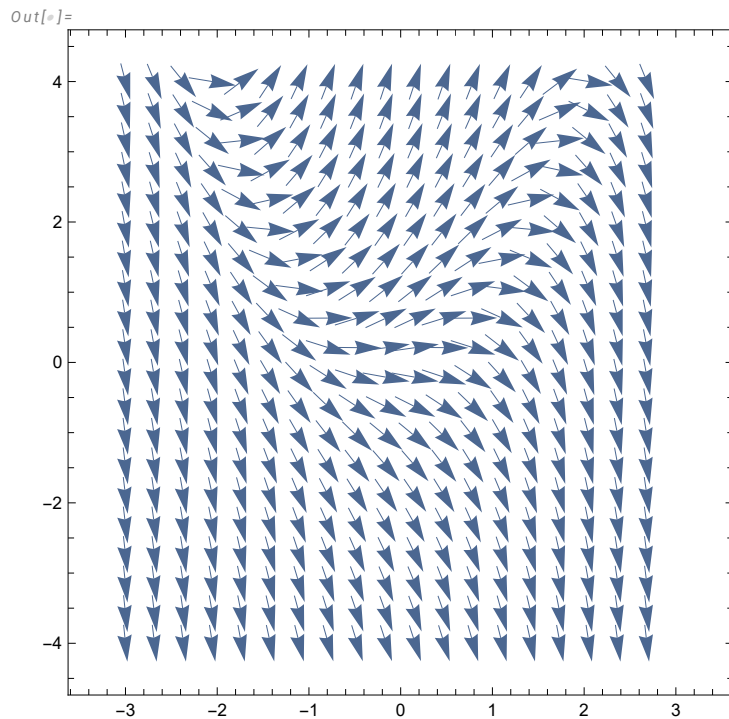
```
Out[ ]:=
```



```
In[ ]:=  
VectorPlot[{1, y - x^2} / (0.1 + 1^2 + (y - x^2)^2)^(1/2),  
{x, -3, 3}, {y, -4, 4}, VectorPoints -> 20]
```



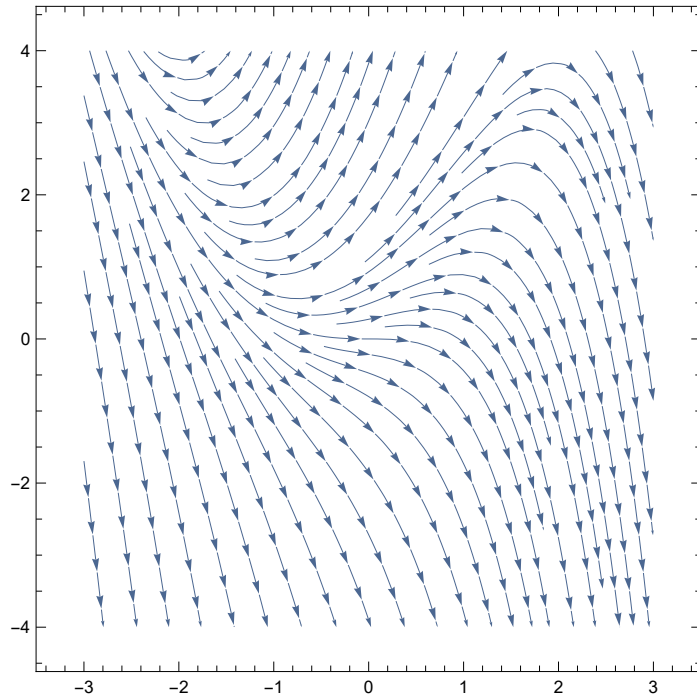
```
In[ ]:=  
VectorPlot[Normalize[{1, y - x^2}], {x, -3, 3}, {y, -4, 4}, VectorPoints -> 20]
```



In[ ]:=

```
StreamPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```

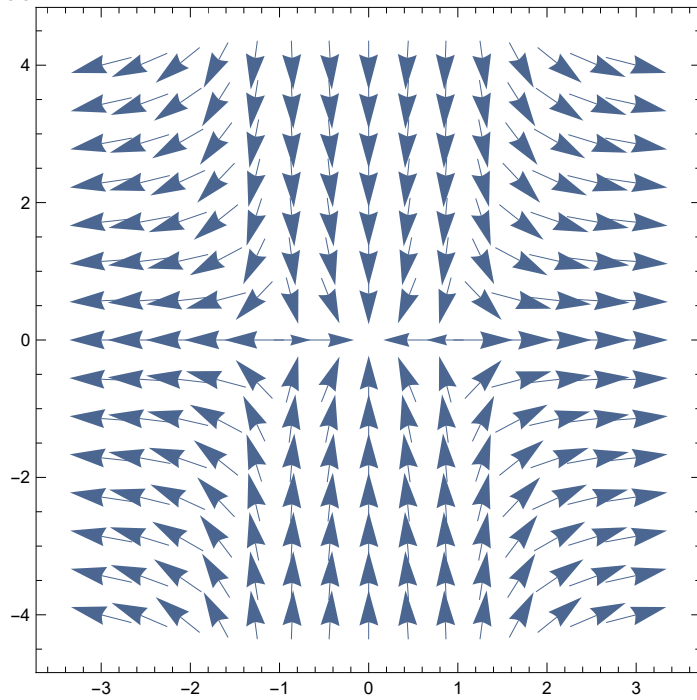
Out[ ]:=



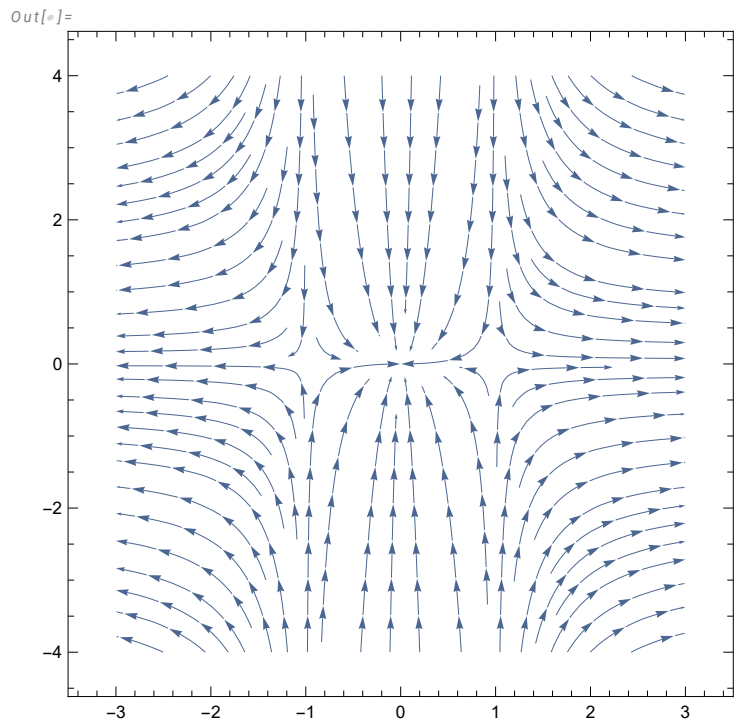
In[ ]:=

```
VectorPlot[{-x + x^3, -2 y} / (0.1 + (-x + x^3)^2 + (-2 y)^2)^(1/2), {x, -3, 3}, {y, -4, 4}]
```

Out[ ]:=



```
In[*]:=
StreamPlot[{-x + x^3, -2 y}, {x, -3, 3}, {y, -4, 4}]
```



```
In[*]:=
DSolve[y' [x] == y[x] - x^2, y[x], x]
```

```
Out[*]=
{{y[x] -> 2 + 2 x + x^2 + e^x c1}}
```

The general solution of the above ODE is You can easily find the above integral using u substitution:  
 $Ce^x + x^2 + 2x + 2.$

```
In[*]:=
DSolve[{y' [x] == y[x] - x^2, y[0] == 2}, y[x], x]
```

```
Out[*]=
{{y[x] -> 2 + 2 x + x^2}}
```

```
In[*]:=
DSolve[{x' [t] == 2 x[t] + y[t], y' [t] == x[t] - 2 y[t]}, {x[t], y[t]}, t]
```

```
Out[*]=
{{x[t] -> (1/10) e^{-sqrt(5) t} (5 - 2 sqrt(5) + 5 e^{2 sqrt(5) t} + 2 sqrt(5) e^{2 sqrt(5) t}) c1 + (e^{-sqrt(5) t} (-1 + e^{2 sqrt(5) t}) c2) / (2 sqrt(5)),
y[t] -> (e^{-sqrt(5) t} (-1 + e^{2 sqrt(5) t}) c1) / (2 sqrt(5)) - (1/10) e^{-sqrt(5) t} (-5 - 2 sqrt(5) - 5 e^{2 sqrt(5) t} + 2 sqrt(5) e^{2 sqrt(5) t}) c2}}
```

In[ ]:=

```
DSolve[{x'[t] == 2 x[t] + y[t], y'[t] == x[t] - 2 y[t], x[0] == 2, y[0] == 1}, {x[t], y[t]}, t]
```


Out[ ]:=

$$\left\{ \left\{ x[t] \rightarrow \frac{1}{2} e^{-\sqrt{5} t} \left( 2 - \sqrt{5} + 2 e^{2 \sqrt{5} t} + \sqrt{5} e^{2 \sqrt{5} t} \right), y[t] \rightarrow \frac{1}{2} e^{-\sqrt{5} t} \left( 1 + e^{2 \sqrt{5} t} \right) \right\} \right\}$$

In[ ]:=

```
NDSolve[{y'[x] == y[x] - x^2, y[0] == 2}, y[x], {x, -3, 3}]
```

Out[ ]:=

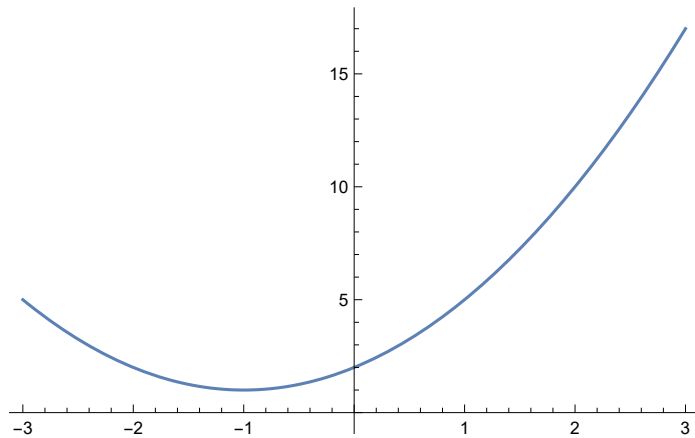
```
{ { y[x] -> InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar ] [x] }
```

The numerical solution can be graphed, as shown below.



In[ ]:=

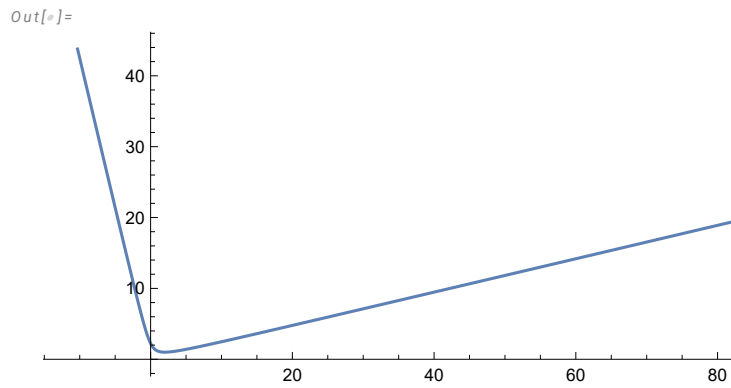
```
Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```

Out[ ]:=

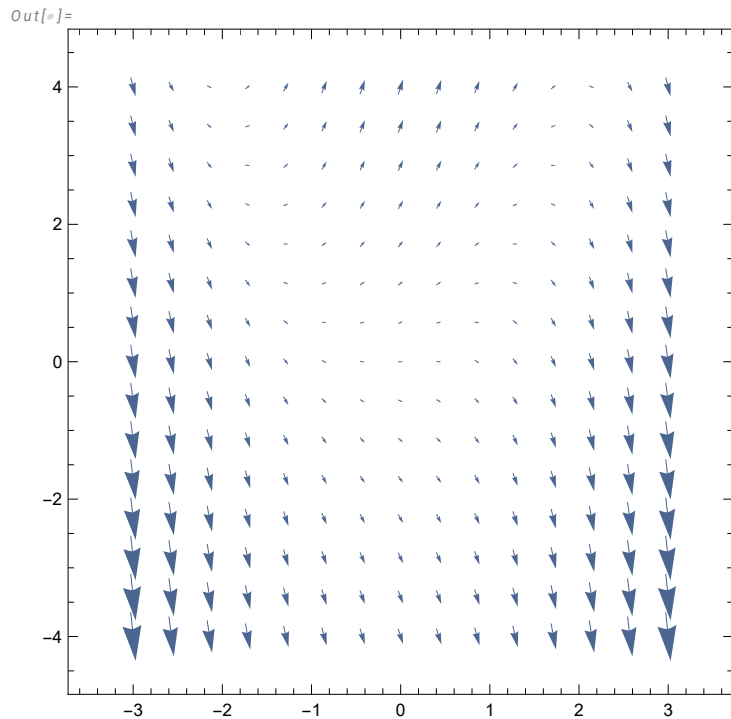


```
In[ ]:=
NDSolve[{x'[t] == 2 x[t] + y[t], y'[t] == x[t] - 2 y[t], x[0] == 2, y[0] == 1},
{x[t], y[t]}, {t, -2, 2}]
ParametricPlot[Evaluate[{x[t], y[t]} /. %], {t, -2, 2}]
```

```
Out[ ]:=
{{x[t] -> InterpolatingFunction[
  {+  Domain: {{-2., 2.}}
  Output: scalar
] [t],
  y[t] -> InterpolatingFunction[
  {+  Domain: {{-2., 2.}}
  Output: scalar
] [t]}}
```




```
In[ ]:=
graph1 = VectorPlot[{1, y - x^2}, {x, -3, 3}, {y, -4, 4}]
```



```
In[ ]:=
NDSolve[{y'[x] == y[x] - x^2, y[0] == 2}, y[x], {x, -3, 3}]
```

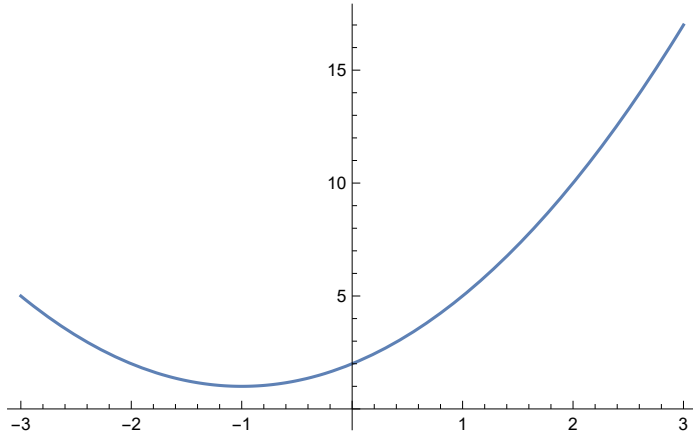
```
Out[ ]:=
```

```
{ {y[x] → InterpolatingFunction[ Domain: {{-3., 3.}} Output: scalar] [x] }
```

```
In[ ]:=
```

```
graph2 = Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```

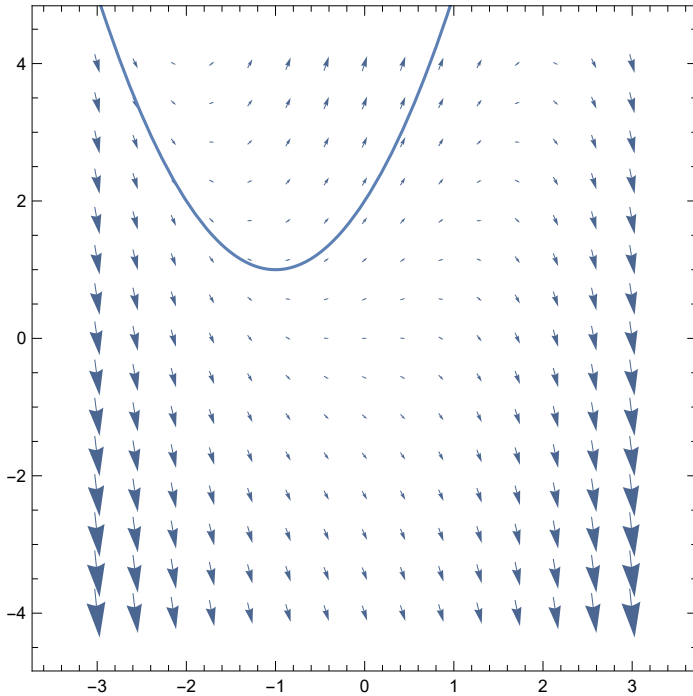
```
Out[ ]:=
```



```
In[ ]:=
```

```
Show[graph1, graph2]
```

```
Out[ ]:=
```

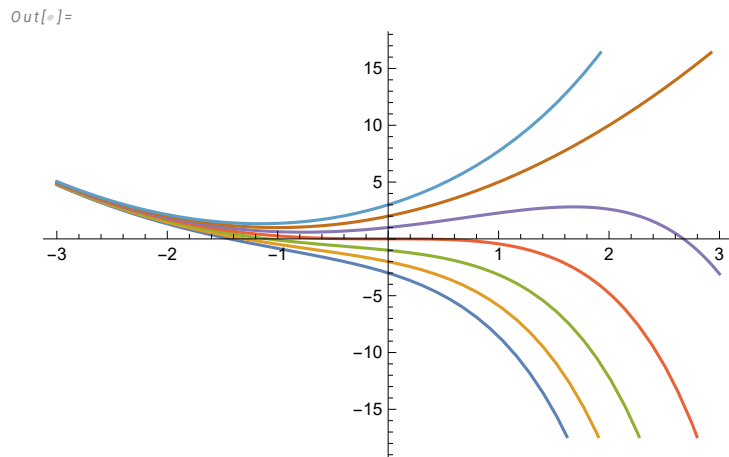


In the above the vector field and one solution are shown together.

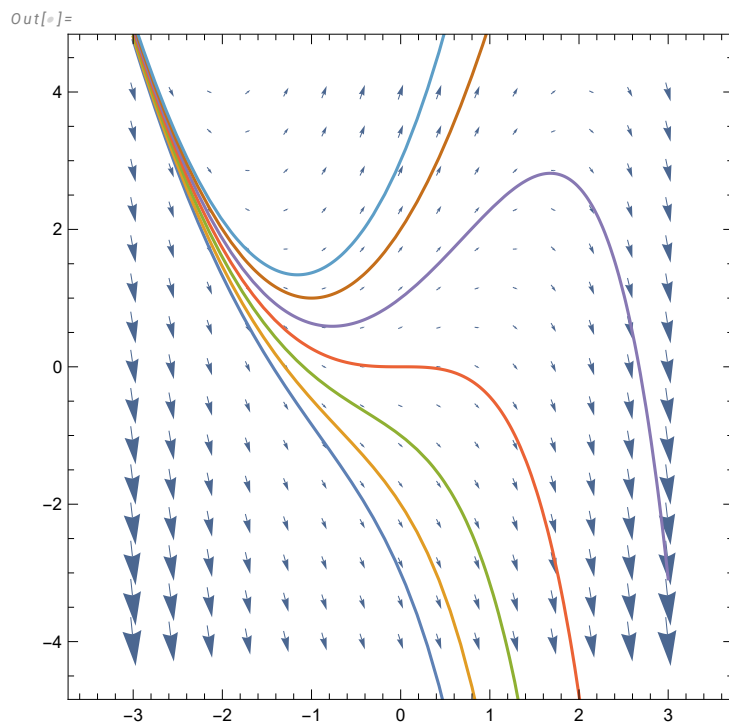




```
In[*]:=
graph3 = Plot[Evaluate[y[x] /. %], {x, -3, 3}]
```



```
In[*]:=
Show[graph1, graph3]
```



In the above the vector field and several solutions are shown together.